Constraining cosmological models using arc statistics in future SZ cluster surveys

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Abstract. Upcoming wide-area surveys in the submillimetre regime will allow the construction of complete galaxy cluster samples through their thermal Sunyaev-Zel'dovich effect. We propose an analytic method to predict the number of gravitationally lensed giant arcs produced by the cluster sample expected to be detectable for the *Planck* satellite. The statistics of lensed arcs can be used to constrain cosmological parameters. This computation implies the choice of an analytic cluster model which has to reproduce the lensing properties of realistic galaxy clusters. Comparing the results of analytic computations and numerical ray-tracing simulations, we show that spherical models with the Navarro-Frenk-White density profile fail by far to produce as many giant arcs as realistic galaxy clusters. This confirms the great importance of asymmetries and substructures in the lensing matter distribution. By elliptically distorting the lensing potential of spherical NFW density profiles, we show that the discrepancy between analytic and numerical computations can almost completely be removed.

1 Introduction

Several studies demonstrated that arc statistics is a very sensitive tool to constrain cosmological models. For example, using ray-tracing simulations and numerically simulated galaxy clusters, Bartelmann et al. [1] showed that the expected number of giant arcs (defined as arcs with length-to-width ratio larger than 10 and B magnitude less than 21.5) changes by orders of magnitude from low density to high density universes, depending also on the value of the cosmological constant. This result, with the exception of the dependency on the cosmological constant, was confirmed by several other authors [4, 5], who used spherical analytic and somewhat idealistic models to describe the cluster lenses and predict the number of giant arcs as a function of the cosmological parameters. Therefore, the comparison of the expectation from such theoretical studies to the observations of strong lensing events in galaxy clusters can be used to constrain the cosmological model.

In order to make such a comparison, a complete sample of galaxy clusters could be selected through the thermal Sunyaev-Zel'dovich effect (hereafter SZ effect). This will be possible in the near future thanks to the upcoming full-sky microwave surveys, like that planned for the *Planck* satellite mission. Although other selection methods could be used for this purpose (i.e. through the X-ray emission from the intra-cluster gas), the SZ selection criterion is much less sensitive to biases due to the dynamical processes in the cluster centre, like cooling flows or merging events.

2 Lensing Sunyaev-Zel'dovich clusters

Upcoming full-sky microwave surveys will detect of order of 10^4 galaxy clusters through their thermal SZ effect. This effect is determined by the Compton-y parameter

$$y(\vec{\theta}) = \frac{k}{m_{\rm e}c^2} \sigma_{\rm T} \int dl \, T(\vec{\theta}, l) \, n_{\rm e}(\vec{\theta}, l) , \qquad (1)$$

where T is the temperature of the ionized intra-cluster gas and $n_{\rm e}$ is the electron number density. The clusters detectable through their thermal SZ effect are those for which

$$Y \equiv \int d^2 \vec{\theta} y(\vec{\theta}) \ge Y_{\min} , \qquad (2)$$

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where the integral is performed over the source area. The lower limit Y_{\min} depends on the angular resolution and the temperature sensitivity of the detector. For example, for the Planck satellite, Y_{\min} is expected to be $\approx 10^{-4}$ arcmin².

As explained in more detail by Bartelmann [3], many SZ clusters will also be efficient weak gravitational lenses. Their weak lensing effects can be quantified using the aperture mass [11],

$$M_{\rm ap}(\theta) = \int d^2 \vec{\vartheta} \kappa(\vec{\vartheta}) U(|\vec{\vartheta}|) , \qquad (3)$$

which is the integral within a circular aperture with radius θ over the convergence κ , weighted by a suitable filter function satisfying the condition

$$\int_0^\theta d^2 \vartheta \vartheta U(\vartheta) = 0. \tag{4}$$

The dispersion of $M_{\rm ap}$ due to the finite number density $n_{\rm g}$ of randomly distributed background galaxies and their intrinsic ellipticity dispersion σ_{ϵ} was computed by Schneider [11],

$$\sigma_M(\theta) = 0.016 \left(\frac{n_{\rm g}}{30 \, \text{arcmin}^2} \right)^{-1/2} \left(\frac{\sigma_{\epsilon}}{0.2} \right) \left(\frac{\theta}{1 \, \text{arcmin}} \right)^{-1} . \tag{5}$$

Clusters produce a significant weak-lensing effect if the signal-to-noise ratio $S = M_{\rm ap}(\theta)/\sigma_M(\theta)$ is larger than a minimal value $S_{\rm min}$. Assuming $S_{\rm min} = 5$, Bartelmann [3] estimated that more than 70% of the original SZ cluster sample expected from *Planck* will be efficient at least for weak lensing, and many of them will also show strong lensing features.

3 Number of arcs

The number of arcs which is expected to be observed in the selected cluster sample can be estimated using the following information:

- both conditions, that clusters must be detectable through the thermal SZ effect and be efficient weak gravitational lenses, give a minimal cluster mass $M_{\min}(z_{\rm L})$ at any given redshift $z_{\rm L}$ [3];
- the number density of dark matter haloes $n_{PS}(M, z_L)$ can be modeled using the Press & Schechter [10] mass function or its generalizations, like the Sheth & Tormen [12] mass function;
- the lensing cross section for arcs with a minimal length-to-width ratio L/W (approximately corresponding to a minimal magnification μ) $\sigma(L/W, \mu; M, z_L, z_S)$ can be computed for any galaxy cluster with mass M at redshift z_L and for any source redshift z_S once the analytic model describing the cluster lens is chosen;
- the source galaxy redshift distribution can be modeled as

$$p(z_{\rm S}) \propto z_{\rm S}^2 \exp(-z_{\rm S}^{\beta}) ,$$
 (6)

where $\beta = 1.5 [13];$

 \bullet finally, the differential number density of sources with a given observed flux F can be assumed to be a power law:

$$\frac{\mathrm{d}n_{\mathrm{S}}}{\mathrm{d}F} \propto F^{-\alpha} \,, \tag{7}$$

where $\alpha \approx 1.8$ in the R-band [13].

Using these ingredients, the optical depth for the formation of arcs with length-to-width ratio larger than L/W is given by

$$\tau(L/W, \mu; z_{\rm S}) = \frac{1}{4\pi D_{\rm S}^2} \int_0^{z_{\rm S}} \int_{M_{\rm min}}^{\infty} \sigma \, dz_{\rm L} \left| \frac{dV}{dz_{\rm L}} \right| n_{\rm PS}(M, z_{\rm L}) (1 + z_{\rm L})^3 dM , \qquad (8)$$

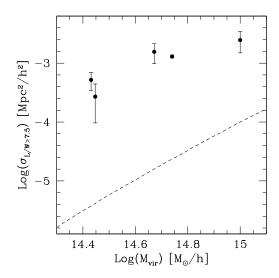


Figure 1: Comparison between numerical and spherical analytic models: the dashed line indicates the cross section for arcs with length-to-width ratio > 7.5 of NFW spheres as a function of the lens mass; filled circles indicate the cross sections of the five numerical clusters used in the ray-tracing simulations.

where $D_{\rm S}$ is angular diameter distance to the sources at redshift $z_{\rm S}$, and ${\rm d}V/{\rm d}z$ is the proper volume per unit redshift.

Finally, the number of arcs with $L/W \ge (L/W)_{\min}$ and magnitude $R \le R_{\max}$ (corresponding to the minimal observed flux F_0) is given by

$$N = \int_0^\infty \int_{\mu_{\min}}^\infty \tau(L/W, \mu; z_{\mathcal{S}}) \left(\int_{F_0}^\infty \frac{\mathrm{d}n_{\mathcal{S}}}{\mathrm{d}(F/\mu)} \frac{\mathrm{d}F}{\mu} \right) p(z_{\mathcal{S}}) \mathrm{d}\mu \mathrm{d}z_{\mathcal{S}} . \tag{9}$$

4 Which analytic model?

The crucial point in this computation is the choice of an analytic model for describing the lenses. Several authors [4, 5] modeled them as isothermal spheres in order to measure their efficiency to produce giant arcs. In this study, we use the much more realistic Navarro-Frenk-White (hereafter NFW) density profile, and we compare the strong lensing properties of this analytic model with those of realistic galaxy clusters obtained from N-body simulations. The NFW profile has strong-lensing properties which differ substantially from the singular isothermal profile [6, 9].

For this purpose, we use a sample of five clusters obtained from dark matter N-body simulations, kindly made available by the GIF collaboration. These are placed at redshift $z_{\rm L}=0.27$ and have virial masses ranging between $\sim 3\times 10^{14}M_{\odot}/h$ and $\sim 10^{15}M_{\odot}/h$. The cosmological model is the $\Lambda{\rm CDM}$ model ($\Omega_{\rm M}=0.3,~\Omega_{\Lambda}=0.7,~h=0.7,~\sigma_8=0.8$). A complete description of the numerical cluster sample can be found in Bartelmann et al. [1]. We use that sample to perform ray-tracing simulations [7, 8], placing a large number of elliptical sources at redshift $z_{\rm S}=1$. Finally, we measure the lensing cross sections for arcs with length-to-width ratio larger than 7.5.

The results are shown in Fig. 1. The plot displays the numerically measured cross sections (filled circles), obtained by averaging the values corresponding to three independent projections of each cluster, as a function of the virial mass of the lens. The error bars show their uncertainties. The dashed line indicates the analytically computed lensing cross section of NFW spheres placed at the same redshift of the simulated clusters. The numerical cross sections are systematically and substantially larger than the analytic ones by roughly two orders of magnitude. This clearly shows that spherically symmetric models, even with a very realistic density profile, fail to reproduce the strong-lensing properties of numerically modeled galaxy clusters.

A possible explanation for this discrepancy is the lack of asymmetries and substructures in the matter distribution of the spherical models. In fact, the importance of both these factors for the strong lensing efficiency of galaxy clusters was demonstrated by Bartelmann et al. [2].

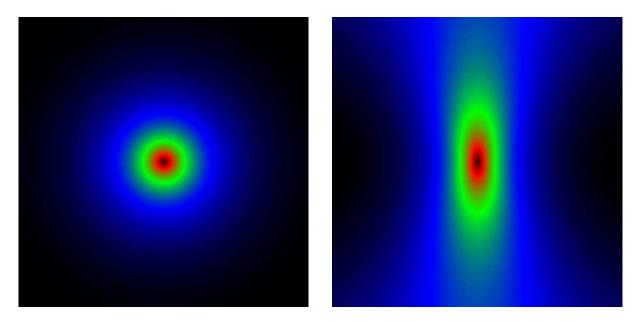


Figure 2: Deflection-angle maps obtained assuming a spherically symmetric NFW lensing potential (left panel), and after adding an ellipticity of e = 0.3 (right panel), as explained in the text.

5 Elliptical NFW lensing potential

The lack of asymmetries and substructures in the matter distribution could be compensated by adding ellipticity to the lensing potential of the NFW spheres.

The spherical NFW lensing potential is given by

$$\Psi(x) \propto \left(\frac{1}{2}\ln^2\frac{x}{2} - 2\operatorname{arctanh}^2\sqrt{\frac{1-x}{1+x}}\right) , \tag{10}$$

where $x = r/r_s$ is the distance from the lens centre r divided by the scale radius r_s of the dark-matter halo

We introduce the ellipticity $e \equiv 1 - b/a$ by substituting

$$x \to \rho = \sqrt{\frac{x_1^2}{(1-e)} + x_2^2(1-e)}$$
 (11)

The components of the deflection angle are then

$$\alpha_1 = \frac{\partial \Psi}{\partial x_1} = \frac{x_1}{(1 - e)\rho} \hat{\alpha}(\rho) \quad ; \quad \alpha_2 = \frac{\partial \Psi}{\partial x_2} = \frac{x_2(1 - e)}{\rho} \hat{\alpha}(\rho) , \qquad (12)$$

where $\hat{\alpha}(\rho)$ is the unperturbed (i.e. spherical) deflection angle at the distance ρ from the lens centre. Using (12), we produce deflection angle maps like those displayed in Fig. 2. The left panel of the figure shows the unperturbed deflection angle map for a lens of $7.5 \times 10^{14} M_{\odot}/h$ at redshift $z_{\rm L} = 0.3$. The right panel shows the map obtained by adding an ellipticity of e = 0.3. Using these deflection-angle maps in the ray-tracing simulations, we measure the lensing cross sections of the elliptical models. These are displayed in Fig. 3 as a function of the ellipticity added. The results show that an ellipticity of e = 0.3–0.4 could suffice to remove the discrepancy between the numerical and the analytic models. Therefore, the usage of elliptical models is more appropriate for computing the number of arcs with a minimal length-to-width ratio produced by a sample of galaxy clusters via gravitational lensing.

6 Conclusions

Upcoming full-sky microwave surveys will detect several thousands of galaxy clusters through their thermal SZ effect. Many of these clusters will also be efficient gravitational lenses.

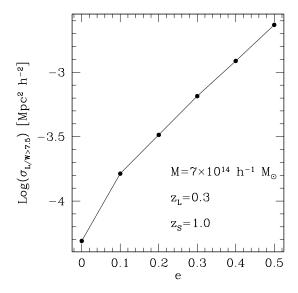


Figure 3: Lensing cross section for arcs with length-to-width ratio larger than 7.5 as a function of the ellipticity added to the lensing potential. The lens mass is $7 \times 10^{14} M_{\odot}/h$, and its redshift is $z_{\rm L} = 0.3$.

We can predict how many *giant* arcs will be produced by this cluster sample by using analytic models to describe the lenses. We verified that spherical models, even with realistic profiles, do not reproduce the lensing properties of numerically modeled galaxy clusters: We need to include at least ellipticity in order to give a correct description of the cluster lenses.

We now plan to calibrate the analytic lensing cross sections for the asymmetric NFW halo profile with numerical cluster models. This can be done by measuring the ellipticity of the lensing potential of the simulated clusters by fitting their deflection angle maps to those of perturbed analytic models.

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